Neutrino number asymmetry and cosmological birefringence

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(Dated: February 1, 2008)

Abstract

We study a new type of effective interactions in terms of the CPT-even dimension-six Chern-Simons-like term, which could originate from superstring theory, to generate the cosmological birefringence. We use the neutrino number asymmetry to induce a sizable rotation polarization angle in the data of the cosmic microwave background radiation polarization. The combined effect of the new term and the neutrino asymmetry provides an alternative way to understand the birefringence.

PACS numbers: 98.80.Cq, 98.80.Es, 11.30.Fs

The polarization maps of the cosmic microwave background (CMB) have been important tools for probing the epoch of the last scattering directly. As we know, the polarization of the CMB can only be generated by Thomson scattering at the last scattering surface and therefore linearly polarized [1, 2]. When a linearly polarized light travels through the Universe to Earth, the angle of the polarization might be rotated by some localized magnetized plasma of charged particles such as ions and electrons, this is so-called Faraday effect. However, the rotated angle of the polarization plane by this Faraday effect is proportional to the square of the photon wavelength and thus it can be extracted.

On the other hand, about ten years ago Nodland and Ralston [3] claimed that they found an additional rotation of synchrotron radiation from the distant radio galaxies and quasars, which is wavelength-independent and thus different from Faraday rotation, referred as the cosmological birefringence. Unfortunately, it has been shown that there is no statistically significant signal present [4, 5]. Nevertheless, this provides a new way to search for new physics in cosmology. Recently, Ni [6] has pointed out that the change of the rotation angle of the polarization can be constrained at the level of 10^{-1} by the data of the Wilkinson Microwave Anisotropy Probe (WMAP) [7] due to the correlation between the polarization and temperature. Feng *et al* [8] have used the combined data of the WMAP and the 2003 flight of BOOMERANG (B03) [10] for the CMB polarization to further constrain the rotation angle and concluded that a nonzero angle is mildly favored. For a more general dynamical scalar, this rotation angle is more constrained [9]. If such rotation angle does exist, it clearly indicates an anisotropy of our Universe. It is known that this phenomenon can be used to test the Einstein equivalence principle as was first pointed out by Ni [11, 12].

Another theoretical origin of the birefringence was developed by Carroll, Field and Jackiw (CFJ) [13]. They modified the Maxwell Lagrangian by adding a Chern-Simons term [13]:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\mathcal{EM}} + \mathcal{L}_{\mathcal{CS}} \\ &= -\frac{1}{4}\sqrt{g}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\sqrt{g}p_\mu A_\nu \tilde{F}^{\mu\nu} , \end{aligned} \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic tensor, $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is the dual electromagnetic tensor, g is defined by g=-det$(g_{\mu\nu})$, and p_ν is a four-vector. Here, to describe a flat, homogeneous and isotropic universe, we use the Robertson-Walker metric

$$ds^2 = -dt^2 + R^2(t)\, d\mathbf{x}^2 , \tag{2}$$

where R is the scale factor; and the totally anti-symmetric tensor Livi-Civita tensor $\epsilon^{\mu\nu\rho\sigma} = g^{-1/2}e^{\mu\nu\rho\sigma}$ with the normalization of $e^{0123} = +1$.

In the literature [4, 13, 22, 23, 24, 25], p_μ has been taken as a constant vector or the gradient of a scalar. In this paper, we study the possibility that the four-vector p_μ is related to a neutrino current

$$p_\mu = \frac{\beta}{M^2} j_\mu \tag{3}$$

with the four-current

$$j_\mu = \bar{\nu}\gamma_\mu \nu \equiv (j_\nu^0, \vec{j_\nu}), \tag{4}$$

where β is the coupling constant of order unity and M is an undetermined new physics mass scale. Note that $\vec{j_\nu}$ is the neutrino flux density and j_ν^0 is the number density difference between neutrinos and anti-neutrinos, given by

$$j_\nu^0 = \Delta n_\nu \equiv n_\nu - n_{\bar{\nu}}, \tag{5}$$

where $n_{\nu(\bar{\nu})}$ represents the neutrino (anti-neutrino) number density. It should be noted that if Δn_ν in Eq. (5) is nonzero, the cosmological birefringence occurs even in the standard model (SM) of particle interactions [14]. However, the effect is expected to be vanishingly small [14]. In the following discussion, we will ignore this standard model effect.

As pointed out by CFJ [13], in order to preserve the gauge invariance we must require that the variation of $\mathcal{L}_{CS}$, given by

$$\mathcal{L}_{CS} = -\frac{1}{2}\sqrt{g}\frac{\beta}{M^2} j_\mu A_\nu \tilde{F}^{\mu\nu}, \tag{6}$$

vanishes under the gauge transformation of $\Delta A = \partial_\nu \chi$ for an arbitrary χ. However, one can check that in general, $\mathcal{L}_{CS}$ in Eq. (6) may not be gauge invariant as

$$\begin{aligned} \Delta\mathcal{L}_{CS} &= \frac{\beta}{4M^2}\chi \tilde{F}^{\mu\nu}(\nabla_\nu j_\mu - \nabla_\mu j_\nu) \\ &= \frac{\beta}{4M^2}\chi \tilde{F}^{\mu\nu}(\partial_\nu j_\mu - \partial_\mu j_\nu), \end{aligned} \tag{7}$$

which does not vanish generally. To achieve the gauge invariance, one could use the Stückelberg formalism[1] [15]. The Lagrangian in Eq. (1) can be reformulated by introducing

[1] We thank Professor R. Jackiw for pointing out this possibility and an encouraging communication.

one Stückelberg field $S^{\mu\nu}$

$$\begin{aligned}\mathcal{L}' &= \mathcal{L}_{\mathcal{EM}} + \mathcal{L}'_{\mathcal{CS}} \\ &= -\frac{1}{4}\sqrt{g}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\sqrt{g}j_\mu(A_\nu\tilde{F}^{\mu\nu} + \partial_\nu S^{\mu\nu})\,, \end{aligned} \tag{8}$$

where $S^{\mu\nu}$ is antisymmetric in indices. It is clear that the requirement of the gauge invariance is easily satisfied by acquiring a gauge transformation of $S^{\mu\nu}$. It is interesting to note that $\mathcal{L}'$ in Eq. (8) might originate from the low energy effective theory in superstring theory[2] in which the role of the Stückelberg field [15, 16] is played by the anti-symmetric Kalb-Ramond field $B_{\mu\nu}$ [17]. For instance, by linking $S^{\mu\beta} = \epsilon^{\mu\beta\sigma\rho}B_{\sigma\rho}$, it is straightforward to show [18] that our effective interaction in Eq. (8) has the same form as Eq. (13.1.42) in Ref. [19]. We remark that the possible superstring origin for $\mathcal{L}_{\mathcal{CS}}$ has also been given in Ref. [20] and the physical effects of the Kalb-Ramond field have been studied in Ref. [20, 21].

As we are working on the usual Robertson-Walker metric, the particle's phase space distribution function is spatially homogeneous and isotropic, i.e. $f(p^\mu, x^\mu)$ reduces to $f(|\,\vec{p}\,|, t)$ or $f(E,t)$ [26]. In other words, the relativistic neutrino background in our Universe is assumed to be homogeneous and isotropic like the CMB radiation, which implies that the number density for neutrinos is only a function of red-shift z, i.e. the cosmic time. As a result, we conclude that the neutrino current in Eq. (4) to a co-moving observer has the form

$$j_\mu = \Big(\Delta n_\nu\big(z(t)\big), \vec{0}\Big)\,. \tag{9}$$

Note that $\vec{j} = -D\vec{\nabla}\big[\Delta n_\nu\big(z(t)\big)\big]$, where D is diffusivity [27] and $\vec{\nabla}$ is the usual differential operators in Cartesian three-space. Here, we have constrained ourselves to consider only the relativistic neutrinos (for homogeneous and isotropic).

From Eq.(9), we have that $\partial_0 j_i = 0$, $\partial_i j_0 = \partial_i n_\nu(z) = 0$ and $\partial_i j_j = 0$. Consequently, we have a curl-free current for the co-moving frame. In this frame, the gauge invariance is maintained and there is no need to include the Stückelberg field. However, the existence of a non-zero component j^0_ν would violate Lorentz invariance [13].

It should be emphasized that the Chern-Simons like term in Eq. (6) is P and C odd but CPT even due to the C-odd vector current of j_μ in Eq. (4), whereas the original one in Ref.

[2] We are very grateful to Dr. W.F. Chen for showing us the string connection as well as sharing his deep mathematical insights.

[13] is CPT-odd [28]. It is clear that $\mathcal{L}_{CS}$ in Eq. (6) is a dimension-6 operator and it must be suppressed by two powers of the mass scale M.

Following Refs. [4, 13], the change in the position angle of the polarization plane $\Delta\alpha$ at redshift z due to our Chern-Simons-like term is given by

$$\Delta\alpha = \frac{1}{2}\frac{\beta}{M^2}\int \Delta n_\nu(t)\frac{dt}{R(t)}\,. \tag{10}$$

To find out $\Delta\alpha$, we need to know the neutrino asymmetry in our Universe, which is strongly constrained by the BBN abundance of ^{4}He. It is known that for a lepton flavor, the asymmetry is given by: [29, 30]

$$\eta_\ell \;=\; \frac{n_\ell - n_{\bar\ell}}{n_\gamma} \;=\; \frac{1}{12\zeta(3)}\left(\frac{T_\ell}{T_\gamma}\right)^3 (\pi^2\xi_\ell + \xi_\ell^3)\,, \tag{11}$$

where n_i $(i = \ell, \gamma)$ are the ℓ flavor lepton and photon number densities, T_i are the corresponding temperatures and $\xi_\ell \equiv \mu_\ell/T_\ell$ is the degeneracy parameter.

As shown by Serpico and Raffelt [29], the lepton asymmetry in our Universe resides in neutrinos because of the charge neutrality, while the neutrino number asymmetry depends only on the electron-neutrino degeneracy parameter ξ_{ν_e} since neutrinos reach approximate chemical equilibrium before BBN [31]. From Eq. (11), the neutrino number asymmetry for a lightest and relativistic, say, electron neutrino is then given by [29, 30, 32]:

$$\eta_{\nu_e} \simeq 0.249\xi_{\nu_e} \tag{12}$$

where we have assumed $(T_{\nu_e}/T_\gamma)^3 = 4/11$. Note that the current bound on the degeneracy parameter is $-0.046 < \xi_{\nu_e} < 0.072$ for a 2σ range of the baryon asymmetry [29, 30]. From Eqs. (5), (11) and (12), we obtain

$$\Delta n_\nu \;\simeq\; 0.061\xi_{\nu_e}T_\gamma^3\,, \tag{13}$$

where we have used $n_\gamma = 2\zeta(3)/\pi^2\; T_\gamma^3$. For a massless particle, after the decoupling, the evolution of its temperature is given by [26]

$$TR \;=\; T_D R_D\,, \tag{14}$$

where T_D and R_D are the temperature and scale factor at decoupling, respectively. In particular, for $R = 1$ at the present time, the photon temperature T'_γ of the red shift z is

$$T_\gamma \;=\; \frac{T_D R_D}{R} = T'_\gamma(1+z)\,. \tag{15}$$

Then, Eq. (10) becomes

$$\Delta\alpha \;=\; \frac{\beta}{M^2} 0.030 \xi_{\nu_e} (T'_\gamma)^3 \int_0^{z_*} (1+z)^3 \frac{dz}{H(z)}\,, \tag{16}$$

where $H(z)$ is given by

$$H(z) = H_0 (1+z)^{3/2} \tag{17}$$

in a flat and matter-dominated Universe and $H_0 = 2.1332 \times 10^{-42} h\ GeV$ is the Hubble constant with $h \simeq 0.7$ at the present. Finally, by taking $1+z_* = (1+z)_{decoupling} \simeq 1100$ at the photon decoupling and $T'_\gamma = 2.73K$, we get

$$\Delta\alpha \;\simeq\; 4.2 \times 10^{-2} \beta \left(\frac{\xi_{\nu_e}}{0.001}\right) \left(\frac{10\,TeV}{M}\right)^2 . \tag{18}$$

As an illustration, for example, by taking $\beta \sim 1$, $M \sim 10\ TeV$ and $\xi_{\nu_e} \sim \pm 10^{-3}$, we get $\Delta\alpha \sim \pm 4 \times 10^{-2}$, which could explain the results in Ref. [8]. We note that a sizable $\Delta\alpha$ could be still conceivable even if the neutrino asymmetry is small. In that case, the scale parameter M has to be smaller.

Finally, we note that there are several other sources which can give rise to this cosmological wavelength-independent birefringence. It is well-known the primordial gravitational vector or tensor perturbations in the CMB could produce a mixtrure of E-mode and B-mode polarizations and generate a non-zero rotation [33, 34]. On the other hand, gravitational lensing also provides a source of the B-mode polarization of the CMB [35]. If there exists a cold neutral dark matter with a non-zero magnetic moment, it will serve as a source of the B-mode CMB polarization and cause a non-zero wavelength-independent rotation angle [36]. In the presence of a quintessence background with a pseudoscalar coupling to electromagnetism, there can also be birefringence by the dynamical quintessence field [37].

In summary, we have proposed a new type of effective interactions in terms of the CPT-even dimension-six Chern-Simons-like term, which could originate from superstring theory, to generate the cosmological birefringence. To induce a sizable rotation polarization angle in the CMB data, a non-zero neutrino number asymmetry is needed. We remark that the Planck Surveyor [38] will reach a sensitivity of $\Delta\alpha$ at levels of $10^{-2} - 10^{-3}$ [12, 39], while a dedicated future experiment on the cosmic microwave background radiation polarization would reach $10^{-5} - 10^{-6}$ $\Delta\alpha$-sensitivity [12].

Note added: After the completion of this work, there was an interesting paper by Cabella, Natoli and Silk [40], which applies a wavelet based estimator on the WIMAP3

TB and EB date to constrain the cosmological birefringence. They derive a limit of $\Delta\alpha = -2.5 \pm 3.0$ deg, which is slightly tighter than that in Ref. [8].

Acknowledgments

We thank Prof. W.T. Ni, Dr. T.C. Yuan, Dr. Y.K. Hsiao and Dr. W. Liao for useful discussions. This work is supported in part by the Natural Science and Engineering Council of Canada and the National Science Council of R.O.C. under Contract #s: NSC-95-2112-M-007-059-MY3 and NSC-96-2918-I-007-010.

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